KIMS-1999-11-12
gr-qc/9911060

Quantum mechanical time contradicts the uncertainty principle

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November 17, 1999

Abstract. The *a priori* time in conventional quantum mechanics is shown to contradict the uncertainty principle. A possible solution is given.

In classical Newtonian mechanics, one can define mean velocity v by $v = x/t$ of a particle that starts from the origin at time $t = 0$ and arrives at position x at time t, if we assume that the coordinates of space and time are given in an *a priori* sense. This definition of velocity and hence that of momentum do not produce any problems, which assures that in classical regime there is no problem in the notion of space-time. Also in classical relativistic view, this would be valid insofar as we discuss the motion of a particle in the coordinates of the observer's.

Let us consider quantum mechanical case where the space-time coordinates are given *a priori*. Then the mean velocity of a particle that starts from a point around the origin at time 0 and arrives at a point around x at time t should be defined as $v = x/t$. The longer the time length t is, the more exact this value will be, if the errors of the positions at time 0 and t are the same extent, say $\delta > 0$, for all t. This is a definition of the velocity, so this must hold in exact sense if the definition works at all. Thus

$$\text{we have a precise value of (mean) momentum } p = mv \text{ at a large time } t \tag{1}$$

with m being the mass of the particle. Note that the mean momentum approaches the momentum at time t when $t \to \infty$ as the interaction of the particle with other particles vanishes as $t \to \infty$.

However in quantum mechanics, the uncertainty principle prohibits the position and momentum from taking exact values simultaneously. For illustration we consider a normalized state ψ such that $\|\psi\| = 1$ in one dimensional case. Then the expectation values of the position and momentum operators $Q = x$ and $P = \frac{\hbar}{i}\frac{d}{dx}$ on the state ψ are given by

$$q = (Q\psi, \psi), \quad p = (P\psi, \psi)$$

respectively, and their variances are

$$\Delta q = \|(Q-q)\psi\|, \quad \Delta p = \|(P-p)\psi\|.$$

Then their product satisfies the inequality

$$\begin{aligned}
\Delta q \cdot \Delta p &= \|(Q-q)\psi\|\|(P-p)\psi\| \geq |((Q-q)\psi, (P-p)\psi)| \\
&= |(Q\psi, P\psi) - qp| \geq |\mathrm{Im}((Q\psi, P\psi) - qp)| \\
&= |\mathrm{Im}(Q\psi, P\psi)| = \left|\frac{1}{2}((PQ - QP)\psi, \psi)\right| \\
&= \left|\frac{1}{2}\frac{\hbar}{i}\right| = \frac{\hbar}{2}.
\end{aligned}$$

Namely

$$\Delta q \cdot \Delta p \geq \frac{\hbar}{2}. \tag{2}$$

This uncertainty principle means that there is a least value $\hbar/2 (> 0)$ for the product of the variances of position and momentum so that the independence between position and momentum is assured in an absolute sense that there is no way to let position and momentum correlate exactly as in classical views.

Applying (2) to the above case of the particle that starts from the origin at time $t = 0$ and arrives at x at time t, we have at time t

$$\Delta p > \frac{\hbar}{2\delta} \tag{3}$$

because we have assumed the error Δq of the coordinate x of the particle at time t is less than $\delta > 0$. But the argument (1) above tells that $\Delta p \to 0$ when $t \to \infty$, contradicting (3).

This observation shows that, if given a pair of *a priori* space and time coordinates, quantum mechanics becomes contradictory.

A possible solution would be to regard the independent quantities, space and momentum operators, as the fundamental quantities of quantum mechanics. As time t can be introduced as a ratio x/v on the basis of the notion of space and momentum in this view[†], time is a redundant notion that should not be given a role independent of space and momentum.

It might be thought that in this view we lose the relation $v = x/t$ that is necessary for the notion of time to be valid, if space and momentum operators are completely independent as we have seen. However there can be found a relation like $x/t = v$ as an approximate relation that holds to the extent that the relation does not contradict the uncertainty principle ([1], [2]).

[†]See [1], [2] for a precise definition.

The quantum jumps that are assumed as an axiom on observation in usual quantum mechanics may arise from the classical nature of time that determines the position and momentum in precise sense simultaneously. This nature of time may urge one to think jumps must occur and consequently one has to observe definite eigenstates. In actuality what one is able to observe is scattering process, but not the eigenstates as the final states of the process. Namely jumps and eigenstates are ghosts arising based on the passed classical notion of time. Or in more exact words, the usual quantum mechanical theory is an overdetermined system that involves too many independent variables: space, momentum, and time, and in that framework time is not free from the classical image that velocity is defined by $v = x/t$.

References

[1] H. Kitada, *Theory of local times*, Il Nuovo Cimento **109 B**, N. 3 (1994), 281-302. (http://xxx.lanl.gov/abs/astro-ph/9309051, http://kims.ms.u-tokyo.ac.jp/time_I.tex).

[2] H. Kitada, *Quantum Mechanics and Relativity — Their Unification by Local Time*, in "Spectral and Scattering Theory," Edited by A.G.Ramm, Plenum Publishers, New York, pp. 39-66, 1998. (http://xxx.lanl.gov/abs/gr-qc/9612043, http://kims.ms.u-tokyo.ac.jp/ISAAC.tex, time_IV.tex).